PMM U.S.S.R.,Vol.44,pp.539-543
0021-8928/81/4 0539 $7.50/0

UDC 539.3

STRESS FIELDS OF DISLOCATION CONFIGURATIONS IN AN ISOTROPIC PLATE*

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A method of determining the stress fields caused by plane dislocation configurations in a plate, is given. Stress fields of dislocation formations with arbitrary Burgers' vectors lying in an arbitrary plane parallel to the plate surface are determined. The problem is solved using a generalization of the method given in /1,2/, and the solution is given in analytic form as well as in a form suitable for numerical computations.

The necessity for the analysis of the elastic stress fields of various dislocation formations in plates arises in connection with extensive practical application of thin films. The problem was studied earlier only for particular cases of infinite rectilinear screw /3/ and edge /4/ dislocations, and of the circular dislocation loops lying in the middle of the plate /1,2/.

1. Formulation of the problem.

Let a dislocation loop of arbitrary shape with arbitrary Burgers' vector b be situated in a plane parallel to the surface of a plate of thickness d. We assume that the plane in which the loop lies is separated from the upper and the lower surface of the plate by the distance h_1 and h_2, respectively. The presence of a dislocation loop implies that a displacement jump

$$\delta \mathbf{u}(\mathbf{r}) = \int\limits_S (\mathbf{b}_\perp + \mathbf{b}_\parallel)\, \delta(\mathbf{r} - \mathbf{r}')\, dS \tag{1.1}$$

exists on the plane S bounded by the line of dislocation. Here $\mathbf{b}_\perp$ and $\mathbf{b}_\parallel$ denote the components of the Burgers' vector of the dislocation loop perpendicular to, and lying in the plane of the loop, and $\mathbf{r}'$ is a variable of integration.

We introduce two Cartesian coordinate systems, the basic (x, y, z) system and the auxilliary (ξ, η, z) system. The xy-plane of the basic system coincides with the plane of the loop. The origin of the auxilliary system is shifted in the xy-plane by a distance denoted by the vector $\mathbf{r}'$. The ξ-axis is directed along the unit vector $\mathbf{k}$ which forms some angle θ with the x-axis.

Using, as the δ-function, its integral representation /5/, we shall write (1.1) in the form

$$\delta \mathbf{u}(\mathbf{r}) = \frac{1}{4\pi^2} \int\limits_S \iint\limits_{-\infty}^{\infty} (\mathbf{b}_\parallel^{(1)} + \mathbf{b}_\parallel^{(2)} + \mathbf{b}_\perp) \cos(\mathbf{k}(\mathbf{r} - \mathbf{r}'))\, d\mathbf{k}\, dS \tag{1.2}$$

Here $\mathbf{b}_\parallel{}^{(1)}$ and $\mathbf{b}_\parallel{}^{(2)}$ are the components of the Burgers' vector $\mathbf{b}_\parallel$ parallel and perpendicular, respectively, to the vector $\mathbf{k}$, $|\mathbf{k}| = \sqrt{l^2 + m^2}$; l and m are the projections of the vector $\mathbf{k}$ on the x- and y-axis respectively. From (1.2) we see that if we know the state of stress $\sigma_{\alpha\beta}{}^{**}$ generated by the Fourier components $\delta u_\xi, \delta u_\eta, \delta u_z$, corresponding to the three components of the Burgers' vector in (1.2)

$$\delta u_\xi = Q,\ \delta u_\eta = \delta u_z = 0;\ \ \delta u_\eta = Q,\ \delta u_\xi = \delta u_z = 0;\ \ \delta u_z = Q,\ \delta u_\xi = \delta u_\eta = 0;\ \ Q = \cos(\mathbf{k}(\mathbf{r} - \mathbf{r}')) = \cos k\xi \tag{1.3}$$

then the stress due to the dislocation loop can be written in integral form

$$(\sigma_{ik}(\mathbf{r}))_\perp = \frac{b_\perp}{4\pi^2} \int\limits_S \iint\limits_{-\infty}^{\infty} \sigma_{ik}^*(\mathbf{k}(\mathbf{r} - \mathbf{r}'))\, d\mathbf{k}\, dS \tag{1.4}$$

$$(\sigma_{ik}(\mathbf{r}))_\parallel = \sigma_{ik}^{(1)}(\mathbf{r}) + \sigma_{ik}^{(2)}(\mathbf{r}),\quad \sigma_{ik}^{(n)}(\mathbf{r}) = \frac{b_\parallel}{4\pi^2} \int\limits_S \iint\limits_{-\infty}^{\infty} \lambda^{(n)} \sigma_{ik}^{*(n)}(\mathbf{k}(\mathbf{r} - \mathbf{r}'))\, d\mathbf{k}\, dS,\quad n = 1, 2,\quad \lambda^{(1)} = \cos(\widehat{\mathbf{k}\mathbf{b}_\parallel}),\quad \lambda^{(2)} = \sin(\widehat{\mathbf{k}\mathbf{b}_\parallel}) \tag{1.5}$$

$$\sigma_{ik}^* = \alpha_{i\alpha}\alpha_{k\beta}\sigma_{\alpha\beta}^{**} \tag{1.6}$$

Here α_{ik} denote the matrix defining the passage from the (ξ, η, z) coordinate system attached to the vector $\mathbf{k}$, to the basic (x, y, z) system. The integrand expressions under the integral over S are integral representations of the Green's function for the stresses caused by the plane dislocation configuration in the plate. The stresses caused by the dislocation loop with an arbitrary Burgers' vector are found by summing the expressions (1.4) and (1.5).

*Prikl.Matem.Mekhan,44,No.4,761-767,1980

Thus the solution of the problem is reduced to finding a stress field in a plate with free surfaces defined by the normal n, provided that the following conditions hold on the surface

$$\langle\sigma\rangle\, n = 0 \tag{1.7}$$

where $\langle\sigma\rangle$ is the dislocation loop stress tensor.

The above particular problems were solved separately for the upper and lower part of the plate. Matching the solutions obtained was carried out by assuming that a displacement jump of the type (1.3) should exist in the plane of the loop $z = 0$. The displacements must correspond to the Fourier components and the stress tensor components must be continuous in the plane $z = 0$.

The first type of displacements (1.3) corresponds to the case of plane deformations. Using the boundary conditions (1.7) and the matching conditions corresponding to this type of displacements, we obtain the solution of the problem with the help of Airy functions in the form

$$\begin{aligned}
&\sigma_{\xi z}^{**\pm} = \mp \gamma f_0^{\pm} \sin k\xi, \quad \sigma_{zz}^{**\pm} = \gamma f_1^{\pm} \cos k\xi \\
&\sigma_{\xi\xi}^{**\pm} = -\gamma f_2^{\pm} \cos k\xi, \quad \sigma_{\eta\eta}^{**\pm} = \nu\,(\sigma_{\xi\xi}^{**\pm} + \sigma_{zz}^{**\pm}), \quad \sigma_{\eta z}^{**} = \sigma_{\xi\eta}^{**} = 0 \\
&\gamma = Gk\,/\,(1-\nu); \quad f_n^{\pm} = f_n\,(k, z, p^{\pm}, q^{\pm}), \quad n = 0, 1, 2 \\
&f_n = M\,[\varphi_n \operatorname{ch}(k\,|z|) + \psi_n \operatorname{sh}(k\,|z|)] \\
&\varphi_0 = (1-\nu)\,[(B_2 - A_2)\,(k\,|z| - C_1/2) + {}^1\!/_2\, C_2\,(B_1 - A_1 + \\
&\quad k\,|z|\,D_1/2)] + k\,|z|\,A_1\,(B_2 - \nu A_2) \\
&\psi_0 = (1-\nu)\,[(B_2 - A_2) - {}^1\!/_2\, C_2\,(k\,|z|\,A_1 - D_1/2)] + (B_2 - \nu A_2)\,\times \\
&\quad (B_1 + A_1 - k\,|z|\,C_1/2) - A_1\,[\nu B_2 + (1-2\nu)\,A_2] \\
&\varphi_1 = (1-\nu)\,k\,|z|\,A_1 C_2/2 + (B_2 - \nu A_2)\,(k\,|z|\,C_1/2 - B_1) + A_1\,\times \\
&\quad [\nu B_2 + (1-2\nu)\,A_2] \\
&\psi_1 = (1-\nu)\,[{}^1\!/_2 C_2\,(B_1 + k\,|z|\,D_1/2) + k\,|z|\,(B_2 - A_2)] - \\
&\quad (C_1/2)\,[\nu B_2 + (1-2\nu)\,A_2] - k\,|z|\,A_1\,(B_2 - \nu A_2)] \\
&\varphi_2 = (1-\nu)\,[2\,(B_2 - A_2) - {}^1\!/_2\, C_2\,(k\,|z|\,A_1 - D_1)] + (B_2 - \nu A_2)\,\times \\
&\quad (B_1 + 2A_1 - 1/2\,k\,|z|\,C_1) - A_1\,[\nu B_2 - (1-2\nu)\,A_2] \\
&\psi_2 = (1-\nu)\,[k\,|z|\,(B_2 - A_2) - {}^1\!/_2\, C_2\,(B_1 - 2A_1 + k\,|z|\,D_1/2)] + \\
&\quad k\,|z|\,A_1\,(B_2 - \nu A_2) - {}^1\!/_2\, C_1\,[(2-\nu)\,B_2 - A_2] \\
&A_1 = \operatorname{sh}^2 kp, \quad A_2 = \operatorname{sh}^2 kq, \quad B_1 = k^2 p^2, \quad B_2 = k^2 q^2 \\
&C_1 = \operatorname{sh} 2kp + 2kp, \quad C_2 = \operatorname{sh} 2kq + 2kq \\
&D_1 = \operatorname{sh} 2kp - 2kp, \quad D_2 = \operatorname{sh} 2kq - 2kq \\
&M = 2\,[C_1\,(B_2 + (1-2\nu)\,A_2) + C_2\,(B_1 + (1-2\nu)\,A_1)]^{-1} \\
&p^+ = h_1, \quad q^+ = h_2, \quad p^- = h_2, \quad q^- = h_1
\end{aligned} \tag{1.8}$$

Here G is the shear modulus and ν is the Poisson's ratio. The quantities with the plus and minus signs refer to the regions $z \geqslant 0$ and $z \leqslant 0$, respectively.

The second type of displacements (1.3) also represent a case of plane deformation. It has the associated stress field

$$\begin{aligned}
&\sigma_{\xi z}^{**\pm(1)} = \gamma g_0^{\pm} \cos k\xi, \quad \sigma_{zz}^{**\pm(1)} = \mp \gamma g_1^{\pm} \sin k\xi \\
&\sigma_{\xi\xi}^{**\pm(1)} = \mp \gamma\, g_2^{\pm} \sin k\,\xi, \quad \sigma_{\eta\eta}^{**+(1)} = \nu\,(\sigma_{\xi\xi}^{**\pm(1)} + \sigma_{zz}^{**\pm(1)}) \\
&\sigma_{\eta\xi}^{**(1)} = \sigma_{\eta z}^{**(1)} = 0, \quad g_n^{\pm} = g_n\,(k, z, p^{\pm}, q^{\pm}), \quad n = 0, 1, 2 \\
&g_n = N\,[\Phi_n \operatorname{ch}(k\,|z|) + \Psi_n \operatorname{sh}(k\,|z|)] \\
&\Phi_0 = (1-\nu)\,A_1\,(A_2 - B_2 - k\,|z|\,D_2/2) + (B_2 - \nu A_2)\,(A_1 - B_1 - \\
&\quad k\,|z|\,D_1/2) \\
&\Psi_0 = (1-\nu)\,[{}^1\!/_2 D_2\,(k\,|z|\,C_1/2 - B_1) + k\,|z|\,(B_2 - A_2)] + \\
&\quad k\,|z|\,A_1\,(B_2 - \nu A_2) - {}^1\!/_2\, D_1\,[\nu B_2 + (1-2\nu)\,A_2] \\
&\Phi_1 = (1-\nu)\,[(B_2 - A_2)\,(k\,|z| + D_1/2) - {}^1\!/_2\, D_2\,(B_1 - A_1 - k\,\times \\
&\quad |z|\,C_1/2)] + k\,|z|\,A_1\,(B_2 - \nu A_2) \\
&\Psi_1 = -(1-\nu)\,[{}^1\!/_2\, D_2\,(k\,|z|\,A_1 + C_1/2) + (1 - A_1)\,(B_2 - A_2)] + \\
&\quad (B_2 - \nu A_2)\,(B_1 + k\,|z|\,D_1/2) \\
&\Phi_2 = (1-\nu)\,[{}^1\!/_2 D_2\,(B_1 + A_1 - k\,|z|\,C_1/2) - (B_2 - A_2)\,(k\,|z| + \\
&\quad D_1/2)] + (B_2 - \nu A_2)\,(D_1 - k\,|z|\,A_1) \\
&\Psi_2 = (1-\nu)\,[(1 - A_1)\,(A_2 - B_2) - {}^1\!/_2\, D_2\,({}^1\!/_2\, C_1 - k\,|z|\,A_1)] \\
&\quad (B_2 - \nu A_2)\,(B_1 - 2A_1 + k\,|z|\,D_1/2) \\
&N = 2\,[D_1\,(B_2 + (1-2\nu)\,A_2) + D_2\,(B_1 + (1-2\nu)\,A_1)]^{-1}
\end{aligned} \tag{1.9}$$

The stress field corresponding to the third type of displacements (1.3) produces antiplane deformation. In this case the equation of equilibrium assumes the form of a Laplace equation, the solution of which has the form

$$\begin{aligned}
&\sigma_{\xi\eta}^{**\pm(2)} = \mp {}^1\!/_2\, Gkg_3\,(k, z, p^{\pm}, q^{\pm}) \sin k\xi \\
&\sigma_{\eta z}^{**\pm(2)} = -{}^1\!/_2 Gkg_4\,(k, z, p^{+}, q^{\pm}) \cos k\xi, \quad \sigma_{\xi\xi}^{**(2)} = \sigma_{\eta\eta}^{**(2)} = \sigma_{zz}^{**(2)} = \sigma_{\xi z}^{**(2)} = 0
\end{aligned} \tag{1.10}$$

$$g_3 = \omega\,[\operatorname{ch}(k\,|\,z\,|) - \operatorname{th}(kp)\operatorname{sh}(k\,|\,z\,|)]$$

$$g_4 = \omega\,[\operatorname{ch}(k\,|\,z\,|)\operatorname{th}(kp) - \operatorname{sh}(k\,|\,z\,|)]$$

$$\omega = 2\operatorname{th}(kp)\,/\,[\operatorname{th}(kp) + \operatorname{th}(kq)]$$

The knowledge of the stress fields (1.8)—(1.10) associated with the Fourier components of the displacements (1.3) enables us to determine, using the formulas (1.4), (1.5) and (1.6), the stress field in a plate for arbitrary dislocations with arbitrary Burgers' vector. Making h_1 or h_2 tend to infinity, we can also determine the stress fields for the dislocation of arbitrary form, with Burgers' vectors, in a semi-infinite medium.

2. **Stress field of the circular disloaction loops in a plate.** Let us consider a solution of the problem for a particular case of a circular dislocation loop of radius a, with an arbitrary Burgers' vector $\mathbf{b}$. Let (x, y, z) and (r, α, z) be the Cartesian and cylindrical coordinate systems with origins at the center of the loop. In this case $n = k\cos\theta$, $m = k\sin\theta$, $r = (r, \alpha)$, $r' = (r', \alpha')$. Integrating (1.2) with respect to ds, we obtain

$$\delta\mathbf{u} = \frac{a}{2\pi}\int_0^{2\pi}\int_0^{\infty}\mathbf{b}\cos(kr\sin\theta)\,J_1(ka)\,dk\,d\theta \tag{2.1}$$

Here J_1 is a Bessel function of the first order. Let us first consider the peripheral dislocation loop. In this case the determination of σ_{ik} is reduced, in accordance with (2.1), to computing the integral

$$\sigma_{ik} = \frac{ab_\perp}{2\pi}\int_0^{\infty}\int_0^{2\pi} J_1(ma)\,\sigma_{ik}^{*}\,dm\,d\theta \tag{2.2}$$

Computing the integral (2.2) with (1.6) and (1.8) taken into account, we obtain the following expression for the components of the elastic stress field of the peripheral loop in the plate:

$$\sigma_{xz}^{\pm} = \mp\beta_\perp F_0^{\pm}\cos\alpha,\quad \sigma_{yz}^{\pm} = \mp\beta_\perp F_0^{\pm}\sin\alpha \tag{2.3}$$

$$\sigma_{xx}^{\pm} = -\beta_\perp\{\nu F_1^{\pm}\sin^2\alpha + F_2^{\pm}[1 - (1-\nu)\sin^2\alpha] + F_3^{\pm}r^{-1}\cos 2\alpha\}$$

$$\sigma_{yy}^{\pm} = -\beta_\perp\{\nu F_1^{\pm}\cos^2\alpha + F_2^{\pm}[1 - (1-\nu)\cos^2\alpha] - F_3^{\pm}r^{-1}\cos 2\alpha\}$$

$$\sigma_{xy}^{\pm} = -\beta_\perp\{-{}^1/_2\,\nu F_1^{\pm} + {}^1/_2\,(1-\nu)F_2^{\pm} + F_3^{\pm}r^{-1}\}\sin 2\alpha$$

$$\sigma_{zz}^{\pm} = -\beta_\perp F_1^{\pm},\quad \beta_\perp = Gb_\perp a\,(1-\nu)^{-1}$$

$$F_0^{\pm} = \int_0^{\infty} kJ_1(ka)\,J_1(kr)\,f_0(k, z, p^{\pm}, q^{\pm})\,dk$$

$$F_i^{\pm} = \int_0^{\infty} kJ_1(ka)\,J_1(kr)\,f_i(k, z, p^{\pm}, q^{\pm})\,dk,\quad i = 1, 2$$

$$F_3^{\pm} = \int_0^{\infty} J_1(ka)\,J_1(kr)\,f_3(k, z, p^{\pm}, q^{\pm})\,dk$$

$$f_3^{\pm} = \nu f_1(k, z, p^{\pm}, q^{\pm}) - (1-\nu)\,f_2(k, z, p^{\pm}, q^{\pm})$$

Figures 1 and 2 depict the character of variation in the shear stresses σ_{xz} in units of $Gb_\perp/(1-\nu)\,a$ at the points in the xz-plane for $\nu = 0.34$, for the cases $2h_1 = h_2 = 0.8\,a$ and $h_1 = h_2 = a$, respectively.

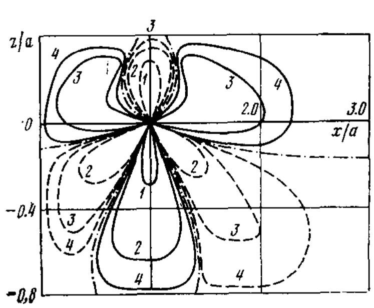

Fig.1

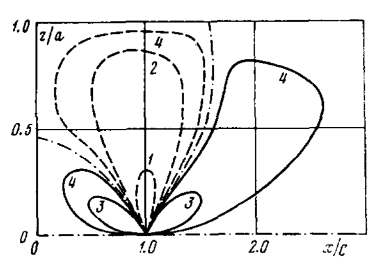

Fig.2

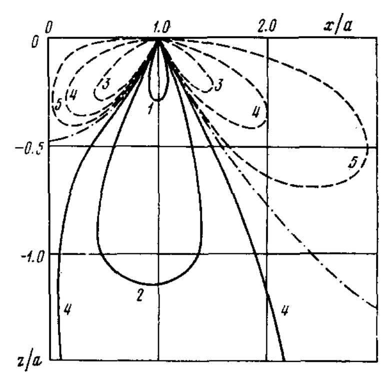

Fig. 3

Figure 3 shows, for comparison, analogous curves for a peripheral loop in an infinite medium.

Curves *1–5* correspond to the values $(1-\nu)\, a\sigma_{xz} / Gb_{\|} = 0.5, 0.1, 0.05,\ 0.02$ and 0.005. The solid, dashed and dot-dash lines denote, respectively, the positive, negative and zero levels of the shear strength. We see that the displacement of the dislocation loop from the central plane may lead to additional emergence of the lines $\sigma_{xz} = 0$ onto the surface. This means that in the case of a nonsymmetrical distribution of the dislocation loops within the plate, the stresses in the layers near the surface may vary not only in magnitude, but also in sign, i.e. the character of the stress distribution may change. The latter fact can exert a significant influence on the physical processes developing in thin films.

For a slipping, circular dislocation loop in the expression (1.5) assumes the form

$$\sigma_{ik} = \frac{ab_{\|}}{2\pi} \int_0^\infty \int_0^{2\pi} J_1(ka)\, [\sigma_{ik}^{*(1)} \sin(\alpha+\theta) + \sigma_{ik}^{*(2)} \cos(\alpha+\theta)]\, dk\, d\theta$$

Here $\sigma_{ik}^{*(i)}$ are connected with $\sigma_{ik}^{**(i)}$ by the relations (1.6), and $\sigma_{ik}^{**(i)}$ are determined by the expressions (1.9) and (1.10). Performing the integration, we obtain

$$\sigma_{zz}^{\pm} = \mp \beta_{\|} \Phi_1^{\pm} \cos\alpha \tag{2.4}$$

$$\sigma_{yy}^{\pm} = \mp \beta_{\|} \{\cos\alpha\, [{}^3/_4\, \nu\Phi_1^{\pm} + (1+3\nu)\, \Phi_2^{\pm} / 4 - (1-\nu)\, \Phi_3^{\pm} / 4] - {}^1/_4\, \Phi_6^{\pm} \cos 3\alpha\}$$

$$\sigma_{xx}^{\pm} = \mp \beta_{\|} \{\cos\alpha\, [(\nu/4)\, \Phi_1^{\pm} + (3+\nu)\, \Phi_2^{\pm} / 4 - {}^1/_4\, (1-\nu)\, \Phi_3^{\pm}] + {}^1/_4\, \Phi_6^{\pm} \cos 3\alpha\}$$

$$\sigma_{xy}^{\pm} = \mp \beta_{\|} \{\sin\alpha\, [-{}^1/_4\, \nu\Phi_1^{\pm} + {}^1/_4\, (1-\nu)\, \Phi_2^{\pm} + {}^1/_4\, (1-\nu)\, \Phi_3^{\pm}] + {}^1/_4\, \Phi_6^{\pm} \sin 3\alpha\}$$

$$\sigma_{xz}^{\pm} = -\beta_{\|} \{\cos 2\alpha\, ({}^1/_2\, \Phi_5^{\pm} - r^{-1} \Phi_7^{\pm}) + {}^1/_2\, \Phi_5^{\pm} + {}^1/_2\, (1-\nu)\, \Phi_4^{\pm}\}$$

$$\sigma_{yz}^{\pm} = -\beta_{\|} \sin 2\alpha\, ({}^1/_2\, \Phi_5^{\pm} - r^{-1} \Phi_7^{\pm}),\quad \beta_{\|} = Gb_{\|}\, a / (1-\nu)$$

$$\Phi_i^{\pm} = \int_0^\infty kJ_1(ka)\, J_1(kr)\, g_i(k, z, p^{\pm}, q^{\pm})\, dk, \quad i = 1, 2, 3$$

$$\Phi_j^{\pm} = \int_0^\infty kJ_1(ka)\, J_0(kr)\, g_j(k, z, p^{\pm}, q^{\pm})\, dk,\ j = 4, 5$$

$$\Phi_6^{\pm} = \int_0^\infty kJ_1(ka)\, J_3(kr)\, [\nu g_1 - (1-\nu)(g_2 - g_3)]\, dk$$

$$\Phi_7^{\pm} = \int_0^\infty J_1(ka)\, J_1(kr)\, g_5(k, z, p^{\pm}, q^{\pm})\, dk$$

$$g_5 = g_0(k, z, p^{\pm}, q^{\pm}) - (1-\nu)\, g_4(k, z, p^{\pm}, q^{\pm}) / 2$$

3. Stress field of the rectilinear dislocations and of other plane dislocation configurations in a plate. Relations (2.3) and (2.4) offer the means of determining the stress fields of rectilinear dislocations parallel to the plate surface. To do this, it is sufficient to shift the coordinate system from the center of the loop to the line of dislocation and carry out the passage to the limit, as $a \to \infty$, in the resulting expressions. For an edge dislocation parallel to the x-axis the slip plane of which is perpendicular to the plate surface, we obtain

$$\sigma_{yz}^{\pm} = \mp \beta_{\perp}^{*} \int_0^\infty f_0(k, z, p^{\pm}, q^{\pm}) \cos ky\, dk \tag{3.1}$$

$$\sigma_{yy}^{\pm} = \beta_{\perp}^{*} \int_0^\infty f_2(k, z, p^{\pm}, q^{\pm}) \sin ky\, dk,\quad \sigma_{zz}^{\pm} = \beta_{\perp}^{*} \int_0^\infty f_1(k, z, p^{\pm}, q^{\pm}) \sin ky\, dk$$

$$\sigma_{xx}^{\pm} = \nu\, (\sigma_{yy}^{\pm} + \sigma_{zz}^{\pm}),\ \sigma_{xz} = \sigma_{xy} = 0,\ \beta_{\perp}^{*} = Gb_{\perp} / \pi(1-\nu)$$

For the edge dislocation oriented along the x-axis, with the slip plane parallel to the plate surface, we have

$$\sigma_{yz}^{\pm} = \beta_{\|}^{*} \int_0^\infty \sin ky \left[g_5(k, z, p^{\pm}, q^{\pm}) + \frac{(1-\nu)}{2}\, g_4(k, z, p^{\pm}, q^{\pm}) \right] dk \tag{3.2}$$

$$\sigma_{zz}^{\pm} = \mp \beta_{\|}^{*} \int_0^\infty g_1(k, z, p^{\pm}, g^{\pm}) \cos ky\, dk$$

$$\sigma_{yy}^{\pm} = \mp \beta_{\|}^{*} \int_0^\infty g_2(k, z, p^{\pm}, q^{\pm}) \cos ky\, dk$$

$$\sigma_{xx}^{\pm} = \nu\, (\sigma_{yy}^{\pm} + \sigma_{zz}^{\pm}),\ \sigma_{xz}^{\pm} = \sigma_{xy}^{\pm} = 0,\ \beta_{\|}^{*} = Gb_{\|} / \pi(1-\nu)$$

For a screw dislocation parallel to the x-axis the results are

$$
\begin{aligned}
&\sigma_{xy}{}^{\pm} = \mp Gb_{\parallel}\,(2d)^{-1}T_1\,/\,T_2, \quad \sigma_{xz}{}^{\pm} = Gb_{\parallel}\,(2\,d)^{-1}T_3\,/\,T_2 \\
&\sigma_{yz}{}^{\pm} = \sigma_{zz}{}^{\pm} = \sigma_{xx}^{\pm} = \sigma_{yy}{}^{\pm} = 0 \\
&T_1 = \mathrm{ch}\,\frac{\pi y}{d}\sin\frac{\pi q^{\pm}}{d}\cos\frac{\pi(p^{\pm}-z)}{d} + \frac{1}{2}\sin\frac{2\pi q^{\pm}}{d} \\
&T_2 = \left(\mathrm{ch}\,\frac{\pi y}{d} - \cos\frac{\pi z}{d}\right)\left(\mathrm{ch}\,\frac{\pi y}{d} + \cos\frac{\pi(q^{\pm}-p^{\pm}+z)}{d}\right) \\
&T_3 = \mathrm{sh}\,\frac{\pi y}{d}\sin\frac{\pi q^{\pm}}{d}\sin\frac{\pi(p^{\pm}-z)}{d}
\end{aligned}
\tag{3.3}
$$

Combining (3.1), (3.2) and (3.3) we can find the stress field of the rectilinear dislocations with arbitrary Burgers' vectors, and the stress fields of any dislocation configurations made of rectilinear dislocations, namely the dislocation dipoles, walls, grids of varying structure, etc., in a plate.

REFERENCES

1. CHOU T.Y., ESHELBY J.D., The energy and line tension of a dislocation in a hexagonal crystal. J. Mech. Phys. Solids, Vol.10, No.1, 1962.
2. CHOU Y.T., Energy of circular dislocation loops in thin plates. Acta Metallurg., Vol.11, No.8, 1963.
3. LEIBRIED G., DIETZE H., Zur Theorie der Schraubenversetzung. Z. Phys., Bd 126, H. 10/12, 1949.
4. KROUPA, F. Napěti a deformace, v nekonečném pāsu zpusobenē hranovou dislokaci. Aplikace Matematiky, Vol.4, No.5, 1959.
5. MADELUNG E., Die Mathematischen Hilfsmittel des Physikers. Dover, N.Y. 1943.

Translated by L.K.